THEY’VE GOTTEN THE RIGHT ANSWER! NOW WHAT?

Vicki Jacobs  
*University of North Carolina at Greensboro*

Susan Empson  
*The University of Texas at Austin*

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Collaborators

Rebecca Ambrose
Heather Martin
University of California–Davis

San Diego State University
Randy Philipp         Lisa Lamb
Bonnie Schappelle     Candy Cabral

University of Georgia
Jessica Pierson Bishop

James Madison University
John Siegfried

University of Texas at Austin
Gladys Krause        D’Anna Pynes

University of North Carolina at Greensboro
Naomi Jessup         Amy Hewitt

Other Partners
SRI   Teachers Development Group

STEP
STUDYING TEACHERS’ EVOLVING PERSPECTIVES

Responsive Teaching in Elementary Mathematics

RT/em
What’s all the fuss about a right answer?

- Answer-oriented society
- Prominent role of standardized tests in education
- Correct answers are seen as the single goal of doing mathematics
- Answers do matter but a broader goal of mathematics is to be engaged in rich ways of reasoning
- Broader goal can get lost when we focus only on getting to a correct answer
If you had time to talk with Carla and Rosie, what would you want to discuss?

**Carla** (Grade 2)
Michael has 50 dollars. He buys a new game. Now he has 23 dollars. How much did Michael spend on the new game?

**Rosie** (Grade 5)
It takes $\frac{1}{5}$ of a gallon of paint to paint one doghouse. How many doghouses can you paint with 4 $\frac{3}{5}$ gallons of paint?
Sam (Grade 1): *Melanie has 4 pockets and in each pocket she has 5 rocks. How many rocks does Melanie have?*
What did we learn after Sam gave a correct answer?

- Confirmation of Sam’s strategy
- Link between cubes and problem quantities
- Emerging knowledge of 10s
- Wrote an equation for the problem
- Understanding of the operation of multiplication
- Some strategy flexibility but not yet ready to move to a more efficient strategy
What did Sam experience after he gave a correct answer?

- Verbal explanation of strategy
- Explicit link between cubes & problem quantities
- Validation of strategy (not just answer)
- Highlighting the use of 5+5=10 with maroon cubes
- Equation for the problem
- Discussion of the operation of multiplication
- Exploration of multiple strategies
### After a correct answer

<table>
<thead>
<tr>
<th>Children can...</th>
<th>Teachers can...</th>
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<tbody>
<tr>
<td>• Consolidate their (fragile) understandings</td>
<td>• Decipher children’s (unclear) strategies</td>
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<td>• Move to more sophisticated understandings, try more sophisticated strategies, or make connections to other mathematical ideas</td>
<td>• Explore children’s thinking to identify what they understand well, what understanding is fragile, and what is not yet within their grasp</td>
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*Highlights that a goal of mathematics is to be engaged in rich ways of reasoning*
How common are conversations *after* a correct answer?

- 129 teachers
  - prospective & practicing (K–3) teachers
  - 0 – 33 years of teaching experience
  - 0 – 9 years of professional development focused on children’s mathematical thinking

- Each teacher interviewed 3 children
  (≥ 4 problems per child)

- Goal of interview: explore how each child thinks
  (not a district assessment)
How common are conversations *after* a correct answer?

- 1798 problems
- 1452 problems solved correctly (>80%)

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<td>Asks no questions or shows a strategy</td>
<td>22%</td>
</tr>
<tr>
<td>Asks the child to explain his/her strategy with minimal probing</td>
<td>60%</td>
</tr>
<tr>
<td>Asks questions to explore the child’s thinking</td>
<td>18%</td>
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Toolbox for Teaching Moves After a Correct Answer

MAKING THE MOST OF

STORY PROBLEMS

Honoring students’ solution approaches helps teachers capitalize on the power of story problems. No more elusive train scenarios!

Story problems are an important component of the mathematics curriculum, yet many adults shudder to remember their own experiences with them, often recalling the elusive train problems from high school algebra. In contrast, research shows that story problems can be powerful tools for engaging young children in mathematics, and many students enjoy making sense of these situations (NCTM 2000; NRC 2001). Honoring children’s story problem approaches is of critical importance so that they construct strategies that make sense to them rather than parrot strategies they do not understand.

To explore how teachers can capitalize on the power of story problems, we chose to study...
Toolbox for Teaching Moves

*After a Correct Answer*

- No best move (many productive moves)
- Toolbox *not* a checklist
- Productive moves are in response to what a child says and does
  - Most moves cannot be pre-planned
  - Moves often arise from attending to the details of the child’s strategy
  - Same move can be productive in one situation and not in another
Toolbox for Teaching Moves

**After a Correct Answer**

- Promoting reflection on the strategy the child just completed
- Connecting the child's thinking to symbolic notation
- Encouraging the child to explore multiple strategies and their connections
- Generating follow-up problems linked to the problem the child just completed
Teaching Moves After Sam’s Correct Answer

- Promoting reflection on the strategy Sam just completed
  - Ask Sam to explain his strategy
    "Why don’t you tell me what you did first?"
  - Ask Sam questions to clarify how specific parts of his strategy are linked to quantities in the problem
    "Four what?" (pockets)
  - Ask Sam questions about specific parts of his strategy that hint at an emerging understanding
    “I noticed when you used these sort of maroon colored ones. Tell me about how that worked when you used those.”
Teaching Moves After Sam’s Correct Answer

- Promoting reflection on the strategy Sam just completed
- Connecting Sam’s thinking to symbolic notation
  - Ask Sam to write an equation that goes with the problem
    - “Is there a way that you could write an equation for this problem?”
    - “So 4 x 5 equals 20. What does ‘times’ mean?”
Teaching Moves After Sam’s Correct Answer

- Promoting reflection on the strategy Sam just completed
- Connecting Sam’s thinking to symbolic notation
- Encouraging Sam to explore multiple strategies and their connections
  - Ask Sam to try a second strategy
    - “Is there any way you could have used your groups of 10 to solve it a different way?”
    - “Is there any way you might think about this problem without using cubes?”
Toolbox for Teaching Moves

After a Correct Answer

- Promoting reflection on the strategy Sam just completed
- Connecting Sam’s thinking to symbolic notation
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- Generating follow-up problems linked to the problem the child just completed
Ryan (Grade 5):

*There are 5 pizzas for 8 kids to share equally. How much pizza could each kid get?*
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There are 5 pizzas for 8 kids to share equally. How much pizza could each kid get?
Promoting Reflection on Ryan’s Strategy

• All questions are focused on Ryan’s understanding of the quantity (5/8) that he generated
  o “Is that enough have a whole pizza?”
  o “Is it enough to have half of a pizza?”
  o “So how much more than one half is 5/8?”

• Wait time

• Persistence in questioning
R: They’re going to get more than a half pizza because 4/8 would be 1/2... then if it was 5/8, it’d be more than half.

T: How much more than half?

R: One fraction.

T: One fraction? (Ryan nods) What would that fraction size be?

R: (pause) One. No. One. (long pause)

T: So I heard you say 5/8 is more than 1/2 cause 4/8 is 1/2?

R: Yes.

T: So how much more than 1/2 is 5/8?

R: Oh, one fraction. One. (pause) One. (pause) Uhhh. (pause) One.

T: What do you think?

R: One half or no (pause) one fourth?

T: One fourth more than 5/8? Why?

R: No, 1/8 because if you added 1/8 to 4/8, it would make it 5/8.

T: Nice job hanging in there with that one.
How might you productively extend the conversation?

Carla
Michael has 50 dollars. He buys a new game. Now he has 23 dollars. How much did Michael spend on the new game?

Rosie
It takes 1/5 of a gallon of paint to paint one doghouse. How many doghouses can you paint with 4 3/5 gallons of paint?

\[
\begin{align*}
\text{\[
\begin{array}{c}
\frac{1}{5} \times 6 = 1 \\
\frac{1}{5} \times 4 = \\
\frac{1}{5} \times 5 = 1 \\
\frac{1}{5} \times 10 = 2 \\
\frac{1}{5} \times 5 = \frac{1}{4} + \frac{3}{5} = 4 \frac{3}{5} \\
\frac{1}{5} \times \frac{3}{8} = \frac{3}{5}
\end{array}
\] You can paint 23 doghouses.}
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Michael has 50 dollars. He buys a new game. Now he has 23 dollars. How much did Michael spend on the new game?

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<tr>
<th>Promoting Reflection on Existing Strategy</th>
<th>Tell us how you solved the problem. What are the “dashes” in your tree diagram? Why did you choose to jump by 10 and by 7?</th>
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<tr>
<td>Connecting to Symbolic Notation</td>
<td>Could you write a number sentence that shows how you solved the problem? You got 27 as your answer. Could you write an equation to show where the 27 came from in your work?</td>
</tr>
<tr>
<td>Exploring Multiple Strategies</td>
<td>Is there another way you could solve this problem? You started at 50. Could you solve the problem starting at 23?</td>
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Rosie

It takes $\frac{1}{5}$ of a gallon of paint to paint one doghouse. How many doghouses can you paint with 4 $\frac{3}{5}$ gallons of paint?

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\begin{align*}
\frac{1}{5} \times 6 &= 1 \\
\frac{1}{5} \times 5 &= 1 \\
\frac{1}{5} \times 10 &= 2 \\
\frac{1}{5} \times 5 &= 1 \\
\frac{1}{5} \times \frac{3}{2} &= \frac{3}{5} \\
1 + \frac{3}{5} &= 1 \frac{3}{5}
\end{align*}
\]

You can paint 2 $\frac{3}{5}$ doghouses.
Rosie

*It takes 1/5 of a gallon of paint to paint one doghouse. How many doghouses can you paint with 4 3/5 gallons of paint?*

| Promoting Reflection on Existing Strategy | Walk us through your strategy.  
Tell me what this means again 1/5 × 10 = 2.  
And what’s this mean in our story when you wrote 1/5 × 10 = 2?  
Stay here just so I can understand what this means to you in the story. 1/5 of a gallon of paint...Tell me again why you added these numbers. How did you get the answer of 23 dog houses? |
|------------------------------------------|---|
| Exploring Multiple Strategies           | Questions about crossed out 1/5 × 4  
This was interesting to me. Was it an accident? What was that 4 about? |
Toolbox for Teaching Moves

*After a Correct Answer*

- Promoting reflection on the strategy the child just completed
- Connecting the child’s thinking to symbolic notation
- Encouraging the child to explore multiple strategies and their connections
- **Generating follow-up problems linked to the problem the child just completed**
Daniella (Grade 1): Miguel had 13 cookies and he gives 6 of them to Robert. How many does he have now? (20, 10) (75, 10) (45, 10) (63, 10)
Charlize (Grade 3): If I brought in 5 chocolate bars, and I gave you 3 ½ chocolate bars, how much did I get to keep for myself? If I had 1 ½ chocolate bars, how much more chocolate bar would I need if I wanted to have 3 chocolate bars?
Generating Follow-Up Problems
Linked to the Problem the Child Just Completed

Change numbers

Daniella (Grade 1)
Miguel had 13 cookies and he gives 6 of them to Robert. How many does he have now?
(20, 10) (75, 10) (45, 10) (63, 10)

Change problem type

Charlize (Grade 3)
If I brought in 5 chocolate bars, and I gave you 3 ½ chocolate bars, how much did I get to keep for myself?
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So What?

How can the toolbox help us in thinking about conversations with children?

Useful for teachers

- Working with children *inside* and *outside* of classroom settings

Useful for professional developers/administrators working with teachers

- Need common video or written work to discuss
- Need to push for specific phrasing of questions
What percentage of the 129 teachers took over over the child’s thinking on a regular basis?

*Before a correct answer:* More than one third

*After a correct answer:* Less than 5%

*After a correct answer may be a natural context for teachers to learn to explore and focus on children’s mathematical thinking.*
The Take Aways

• Many benefits of extending conversations after a correct answer
• Many missed opportunities
• Use the tool box to get started
• And a couple of tips!
Thank you!

Enjoy the rest of the conference!