Responsive Teaching in Elementary Mathematics: The Case of Fractions

Susan Empson, The University of Texas at Austin
Vicki Jacobs, University of North Carolina at Greensboro

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• “Effective mathematics teaching elicits evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning”

  – NCTM, *Principles to Action*
• Problem posed without demonstrating a specific strategy
  – To learn about students’ thinking and what they understand

The zoo keeper has 8 bananas to feed to the 6 monkeys. If she wants to use up all the bananas and give the same amount to each monkey, how much should she give each monkey?
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Teaching that is responsive to children’s mathematical thinking

• In-the-moment decisions about what to pursue and how to pursue it, in response to students’ thinking

• **Instructional practices** centered on students’ thinking
<table>
<thead>
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</thead>
<tbody>
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</tr>
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**Responsive Teaching**
Noticing

• Professional noticing of children’s mathematical thinking (Jacobs et al., 2011)
  – Attending to details
  – Interpreting the understanding
  – Deciding how to respond

• Invisible practice

• Drives responsive teaching
Solve it!

On a field trip to the museum, 12 kids were given 16 pizzas to share equally. How much should each kid get?

...the way you think an elementary student might solve it
Knowledge of Children’s Thinking: Types of Strategies

Beginning Understanding: Students are not initially thinking about how to partition the shared items with the number of sharers.

On a field trip to the museum, 12 kids were given 16 pizzas to share equally. How much should each kid get?
Emergent Understanding: Coordination between partitions of shared items with the number of sharers at the beginning of the strategy (additive relationship)

On a field trip to the museum, 12 kids were given 16 pizzas to share equally. How much should each kid get?
Advanced Understanding: Coordination between partitions of shared items with the number of sharers at the beginning of the strategy (multiplicative relationship)

On a field trip to the museum, 12 kids were given 16 pizzas to share equally. How much should each kid get?
Noticing students’ thinking in written work

Take some time to look over the student work from Brandon’s classmates and think about the following questions:

• How does each child solve the problem?
• What does the strategy tell you about the child’s understanding?
• Bonus – What fraction concepts could you address with this student work?
The zookeeper had 8 bananas to feed 6 monkeys. If she wants to use up all of the bananas and give each monkey the same amount, how much should she give each monkey?
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The zookeeper has 8 bananas to feed to the 6 monkeys. If she wants to use up all the bananas and give the same amount to each monkey, how much should she give each monkey? Each monkey can get one whole but you have to split the other 2 and there is six monkeys so they would get \( \frac{1}{2} \) and if you add it together you get \( \frac{1}{2} \). 

\[
\begin{align*}
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The zookeeper has 8 bananas to feed to the 6 monkeys. If she wants to use up all the bananas and give the same amount to each monkey, how much should she give each monkey?

\[ 8 \div 6 = 1 \frac{2}{6} \]

\[ (6 \div 6) + \frac{1}{6} \times 2 = 1 \frac{3}{6} \]

\[ 6 \div 6 + 2 \div 6 = 1 \frac{2}{6} \]
Noticing allows a teacher to...

• Introduce and build conceptual understanding of fractions by building on children’s thinking
  – Fractions as quantities
  – Wholes can be partitioned into parts
  – Parts are related to wholes
  – Fractions are related to division

• Address multiple content standards
### Responsive Teaching

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The diagram illustrates the cyclical nature of responsive teaching, emphasizing the importance of noticing children's mathematical thinking before responding, supporting it before the correct answer, and extending it after the correct answer.
Supporting and extending children’s thinking

• Questioning *before* and *after* a correct answer
The zoo keeper has 8 bananas to feed to the 6 monkeys. If she wants to use up all the bananas and give the same amount to each monkey, how much should she give each monkey?
Some possible questions for Brandon

- How did you solve this problem?
- What are these little squares?
- Where are the last two bananas in your drawing?
- Why did you decide to split the last two bananas each into three parts?
- How big is one of those parts?
  - How much of a banana is it?
  - Is it more or less than ½ banana?
- How much does each monkey get?
- Would you say this is true or false?
  \[ 1\frac{1}{3} = 1 + \frac{1}{3} \]
“Stick with the kid”

These supporting and extending questions are centered on Brandon’s thinking. They...

– Invite him to articulate and reflect on his ideas
– Nudge him to advance his ideas
How does the teacher respond to Ryan’s mathematical thinking?

There are 5 pizzas for 8 kids to share equally. How much pizza could each kid get?
Teaching moves

• Allowed plenty of time for Ryan to think
  – Did not insist that he answer

• In response to initial incorrect response, explored his thinking, rather than redirect it

• Invited Ryan to anticipate final share at a strategic point:
  – Do you know how much each person is going to get?

• Invited Ryan to compare his final share to a benchmark and reason about the relationship between ½ and ⅝
  – Is it enough for each kid to have a whole pizza? How do you know?
  – Is it enough for each kid to have ½ pizza? How do you know?
  – How much more than ½ is ⅝? What would that fraction size be?
Supporting and Extending: *Exploring the details of a child’s strategy*

- Press for detailed explanation (even on incomplete strategies)
- Invite to anticipate next step (at a strategic growth point)
- Invite to reflect on quantities in the solution
  - compare to benchmarks
  - reason about relationships
Supporting and Extending During Instruction

- 4th/5th grade classroom
- Spring of school year
- Instruction:
  - Problems posed
    - Differentiated number selection, problems
  - Teacher circulates
  - Student strategies shared and discussed as group
Video Example
Teaching moves

• Asked a starter question
  – Tell me about your picture

• Invited child to connect representation to quantities in the problem
  – You’re saying that’s how much of that cup she drank? How far has she gone when she drinks that much?
  – Do you want to put something down there so you can keep track of your miles?

• Invited child to reflect on quantities in the strategy
  – Would she have any left?
Supporting and Extending: Exploring the details of a child’s strategy

- Ask a starter question
- Press for detailed explanation (even on incomplete strategies)
- Invite to connect representation to quantities in the problem
- Invite to reflect on quantities in the solution
  - compare to benchmarks
  - reason about relationships
- Invite to anticipate next step
The power of questioning that sticks with the kid