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Responsive Teaching in Elementary Mathematics
What Do We Mean By Noticing?

- Everyday noticing vs. distinct patterns of noticing for groups of professionals (Goodwin, 1994; Mason, 2002; Stevens & Hall, 1998)

- Noticing of mathematics teachers — how do teachers make sense of instructional environments? (Santagata et al., 2007; Star & Strickland, 2007; van Es & Sherin, 2008)
  - What do teachers notice in their classrooms?
  - What do teachers *not* notice?
  - What type of noticing is most productive?
Professional Noticing of Children’s Mathematical Thinking

Three interrelated component skills:
1. **Attending** to children’s strategies
2. **Interpreting** children’s understandings
3. **Deciding how to respond** on the basis of children’s understandings — NOT the actual execution of the response

**When Does Noticing Happen in the Classroom?**

*What goes on behind the scenes before the teacher responds?*

**Child says or does something**

**Teacher responds**
Multiple Types of Teacher Noticing

- Our focus is one type of teacher noticing
  - How and to what extent do teachers notice children’s mathematical thinking?

- Other sample foci
  - Range of things teachers notice
  - Noticing of equitable instructional practices
Studying Teachers’ Evolving Perspectives

- Worked with 129 teachers—prospective and practicing teachers

- Assessed teachers’ noticing with a variety of artifacts
  - Written work
  - Classroom video
  - Video of a 1-on-1 conversation
Sample Prompts for Assessing Professional Noticing of Children’s Mathematical Thinking

1. **Attending** to children’s strategies
   
   Please describe in detail what you think each child did in response to this problem.

2. **Interpreting** children’s understandings
   
   Please explain what you learned about these children’s understandings.

3. **Deciding how to respond** on the basis of children’s understandings
   
   Pretend that you are the teacher of these children. What problem or problems might you pose next? Why?
Percentage of Each Teacher Group Showing Evidence of the Top Level for Each Component Skill

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### Percentage of Each Teacher Group Showing Evidence of the Top Level for Each Component Skill

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<tr>
<td><strong>Attending</strong> to children’s strategies</td>
<td>43%</td>
<td>65%</td>
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<tr>
<td><strong>Interpreting</strong> children’s understandings</td>
<td>0%</td>
<td>16%</td>
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<td><strong>Deciding</strong> how to respond on the basis of children’s understandings</td>
<td>0%</td>
<td>3%</td>
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Rex (Kinder) — How might you respond to Rex?

Rex had 13 cookies. He ate 6 of them. How many cookies does Rex have left?

Today is June 5 and your birthday is June 19. How many days away is your birthday?

Rex had 15 tadpoles. He put 3 tadpoles in each jar. How many jars did Rex put tadpoles in?
“Rex really prefers to use his fingers as a tool to solve problems. In the first problem he used them to count down from 13…In the second problem he counted on from June 5th to June 19th, but was thrown—ever so slightly—when his counting on continued beyond his 10 fingers. Considering this, I think the third problem caused some difficulty because he couldn’t represent 15 tadpoles with his fingers. ... I’d start by asking him why that problem was hard. Is it because of the language and context of tadpoles? Is it because he can’t use a counting on or back strategy? ... Where I’d go from there would really depend on his response…

I might adjust the numbers to (16, 2) to see if he’d skip-count by 2s up to 16 and keep track on his fingers. If Rex explained that it was hard to use his fingers for this one, I might ask if there’s another tool that would help him.”
Describe some ways you might respond to Rex, and explain why you chose those responses.

“I would help him draw a picture and guide him through the problem. I would ask him to draw 15 dots or lines to represent the 15 tadpoles. Then I would tell him that there will be 3 in each jar, so to represent each jar he could circle tadpoles in groups of 3. I would then ask him how many circles he has....”

“I might say something like, ‘Yes, that does seem a little bit harder than our last problems, but you’re a smart boy. I’m sure if we work together, we could solve it.’ ...I’d use positive reinforcement by telling him I think he’s smart to boost his confidence. ...I believe after solving the problem together, Rex would feel very proud of himself.”
So What?

When teachers consistently focus on children’s thinking

Instruction will look different

Instruction is more likely to...

- Honor children’s ways of reasoning
- Highlight the sense making and creativity in mathematics
- Make mathematics more accessible by starting with children’s existing understandings
What did we learn about noticing and the development of noticing expertise?

- Noticing children’s mathematical thinking is complex
- Teachers do not automatically have noticing expertise
- Expertise can be learned, and long-term professional development matters

How do we help teachers gain noticing expertise?

Share glimpse of a new professional development tool we have been piloting...
Responsive Teaching in Elementary Mathematics

- Professional development design experiment
- Grades 3–5 teachers
- Children’s thinking about fractions
- Teachers engage in 3 years of professional development
  - Learning Partners—at least 2 teachers per school
  - Workshops—8.5 days per year (spread across summer & academic year)
  - School-Based Component—engage in 4 Collaborative Inquiry sessions per year to develop noticing expertise
Collaborative Inquiry Tool

- Bridge between workshops and teachers’ classrooms
- Online tool to foster focused, face-to-face conversation at schools
- Teachers work in pairs or small groups
  - Try 1–2 strategically selected problems with their class
  - Bring student written work to teacher-run meeting
  - Analyze strategically selected video or written work
  - Analyze written work from their own classes
- Analysis focused on professional noticing of children’s mathematical thinking — attending, interpreting, and deciding how to respond
Pose these problems to your students before your Collaborative Inquiry session. When you pose the problem, let students know to

- use a strategy that makes sense to them
- use any tool that helps them think, including diagrams, cubes, fingers, number facts, and equations -- anything at all!

You may need to "unpack" the problem context to help students imagine the situation and what it involves. You can have them explain in their own words, visualize the problem, or think about a time they shared something or packed something into boxes.

Problem 1
(Measurement Division) Coach Brown has 56 baseballs. 8 baseballs fit in a box. How many boxes can Coach Brown fill? (126, 10) (180, 12)

Problem 2
(Partitive Division) There are 42 jellybeans in a bag. 7 children want to share them so that they each get the same amount. (122, 10) (250, 12)

Remember To:

- Circulate around the class while students are working on the problems.
- Ask questions to clarify what students are doing and thinking.
- Take notes on students' work with physical objects.
- Collect the written work of six students to bring to the Collaborative Inquiry session.
Third grader Zakyla solved two problems that are similar to the ones you posed for your students. Watch each video and discuss:

- Zakyla's strategy in detail
- What Zakyla understands as revealed by the strategy

Coach Brown has 56 baseballs. 8 baseballs fit in a box. How many boxes can Coach Brown fill? (Measurement Division)

There are 42 jellybeans in a bag. 7 children want to share them so that they each get the same amount. (Partitive Division)
Explore Your Students' Work

Look over the student work you each brought and choose a total of about 3 students to discuss. Make sure that each teacher chooses at least one student from his or her class and that you have a variety of strategies that are correct, so that you can see the range in students' thinking.

Then discuss

1. each student's strategy, in detail
2. what the student understands, as revealed by the strategy

Example: What We Noticed About Zakyla's Thinking

- We noticed that Zakyla solved each problem by direct modeling. Did any of your students use a direct modeling strategy? Did any of your students use strategies that look very different?
- We also noticed that for the jellybeans problem, Zakyla was persistent. She tried a couple of different tools and made an initial guess about how many jelly beans would go to each friend. Did you have any students were persistent in their search for a solution?
Now think about how you can use what you have learned to pose a new problem that builds on students' thinking.

1. Write one follow-up problem for the student you discussed from your own class.
2. Within the next few days, pose the problem to students in your class. Elicit student thinking as you circulate and ask clarifying questions until you understand how students were thinking when they engaged with the problem.

**Example: Deciding How to Respond to Zakyla**

We noticed Zakyla direct modeled each problem but for the jelly bean problem, she skip counted by 5s after distributing 5 jelly beans to each of the 7 children. We wondered if she is ready to move to a counting strategy with familiar numbers like 5. To explore this question, we would pose problems like these:

1. Sandra has 45 candies. She wants to put 5 candies in each goody bag for her birthday party. How many goody bags can she make? *(Measurement Division)*
2. Vanessa got 14 nickels for doing a chore for her mom. How much money did Vanessa get? *(Multiplication)*

We also noticed that Zakyla spent time deciding what tool to use on the jelly bean problem, and she rejected the base-ten blocks because she could not break them apart. We wondered if she would choose to use them to solve a problem if distributing a rod of 10 was a possibility, and whether she would chunk by 10. To explore this question, we would pose problems like these:

1. Dimitri has 42 marbles and 3 friends. He wants to give each friend the same number of marbles, so that each friend gets the same amount. How many marbles can each friend have? *(Partitive Division)*
2. Twelve friends each have 11 stickers. How many stickers do they each have altogether? *(Multiplication)*
Final Thoughts

- Teachers’ noticing of children’s mathematical thinking is worthy of study
  - Theoretical lens
  - Core practice that teachers need to learn (Grossman et al., 2009)

- Hard to study and discuss—hidden practice

- Support for teachers in developing expertise
  - Collaborative Inquiry Tool is one possibility
  - Other suggestions?
Interested in learning more about mathematics teacher noticing?