PROFESSIONAL NOTICING OF CHILDREN’S MATHEMATICAL THINKING: A HIDDEN SKILL OF TEACHING

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What is Noticing?

Everyday use of the term “noticing”

vs.

Noticing done by individuals in a profession

(Goodwin, 1994; Mason, 2002; Stevens & Hall, 1998)

• Noticing of mathematics teachers
  How do teachers make sense of instructional environments?
  (Santagata et al., 2007; Star & Strickland, 2007; van Es & Sherin, 2002, 2008)

• My focus for noticing — how, and to what extent, do teachers notice children’s mathematical thinking? (Jacobs et al., 2010, 2011)

Professional Noticing of Children’s Mathematical Thinking
Collaborators

San Diego State University
Randy Philipp       Lisa Lamb
Bonnie Schappelle  Candy Cabral

Texas State University
Jessica Pierson Bishop

James Madison University
John Siegfried

University of Missouri
Susan Empson

University of Texas at Austin
Gladys Krause       D’Anna Pynes

University of North Carolina at Greensboro
Amy Hewitt           Naomi Jessup

Other Partners
SRI   Teachers Development Group
What does a classroom look like when teachers notice and build on children’s mathematical thinking?

*Responsive Teaching*
(Jacobs & Empson, 2016; Pierson, 2008; Robertson, Scherr & Hammer, 2016)

- Grade 1 class

- *Mrs. Kick bought 4 seed packets. Each packet contains 11 seeds. How many seeds did she buy in all?*

- Launch, Explore, Discuss
Mrs. Kick bought 4 seed packets. Each packet contains 11 seeds. How many seeds did she buy in all?
What does a classroom look like when teachers notice and build on children’s mathematical thinking?

Mrs. Kick bought 4 seed packets. Each packet contains 11 seeds. How many seeds did she buy in all?

- Children’s voices (ideas) dominate
- Multiple strategies
- Sense making
- Mathematical talk
- Children learn from each other

- Teacher notices children’s ideas & builds on them
  - Highlights important mathematical details—asks clarification and why questions
  - Questions cannot be preplanned—teachers must NOTICE the details in what children say and do to generate questions
Why do we care about responsive teaching in which teachers notice and build on children’s thinking?

- Vision of instruction endorsed by policy (CCSSM, 2010; Daro, 2011; NCTM, 2014)

- Higher student achievement & affective benefits (Bobis et al., 2005; Carpenter et al., 1989; Sowder, 2007; Wilson & Berne, 1999)

- Generative for teacher learning (Franke et al., 1998, 2001)

Looking at teachers’ professional noticing helps us decompose responsive teaching (Grossman et al., 2009)
Teachers’ Professional Noticing

- Classrooms are complex environments—teachers cannot attend to everything
- Multiple types of noticing needed

Noticing children’s mathematical thinking is a complex, hidden practice in teaching.

What goes on behind the scenes before the teacher responds?

Child says or does something

Teacher responds
Professional Noticing of Children’s Mathematical Thinking
(Jacobs et al., 2010)

Three interrelated component practices:

1. **Attending** to the details of children’s strategies

2. **Interpreting** children’s understandings reflected in their strategies

3. **Deciding how to respond** on the basis of children’s understandings (*intended response*)

How did we study teacher noticing?
- Video of a classroom lesson
- Video of a 1-on-1 interaction between a teacher and a child
- Set of student written work
Professional Noticing of Children’s Mathematical Thinking

1. **Attending** to the details of children’s strategies

   Please describe in detail what you think each child did in response to this problem.

2. **Interpreting** children’s understandings reflected in their strategies

   Please explain what you learned about these children’s understandings.

3. **Deciding how to respond** on the basis of children’s understandings

   Imagine that you are the teacher of this child. Describe some ways you might respond to his/her work on this problem, and explain why you chose those responses.

   Imagine that you are the teacher of these children. What problem or problems might you pose next? Why?
Development of Professional Noticing Expertise

- Do teachers automatically possess professional noticing expertise?
- Do they gain it with teaching experience?
- Do they need PD?
- If so, how much PD?
129 participants (4 groups, 30+ per group)
  - Prospective elementary school teachers
  - 3 groups of practicing elementary school teachers in grades K–3

Groups of practicing teachers
  - Similar teaching experience (average of 14–16 years per group, none with less than 4 years)
  - Varying amounts of PD (none, 2 years, or ≥ 4 years and leadership activities)

PD focused on children’s mathematical thinking about whole number operations and place value
### Noticing Children’s Mathematical Thinking (Jacobs et al., 2010)

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Development of Professional Noticing Expertise

- Expertise is not automatic but can be learned through sustained PD

- PD beyond 2 years matters
  - *Attending* expertise seemed to develop for most participants during the first 2 years of PD
  - *Deciding How to Respond* expertise developed during the first 2 years of PD and further after 2 years
Professional Noticing Expertise

Deciding How to Respond on the Basis of Children’s Understandings

- What does it look like?
- How is it linked to teachers’ expertise in attending to the details of strategies?

*Imagine that you are the teacher of this child. Describe some ways you might respond to his/her work on this problem, and explain why you chose those responses.*

*Imagine that you are the teacher of these children. What problem or problems might you pose next? Why?*

Characterize the 1-on-1 conversations teachers anticipate having with children about their strategies (e.g., when circulating during problem solving)
Responsive Teaching in Elementary Mathematics

• 4-year professional development design study

• Focus on children’s fraction thinking

• 3 years of PD (total > 150 hours)
  o Summer and academic-year workshops (8.5 days per year)
  o School-based activities
Who participated?
• 72 grades 3–5 teachers (and instructional specialists)
• Range in years of teaching experience & previous PD on children’s mathematical thinking
• Drawn from 3 districts with varying instructional contexts

Today’s Focus: What were the teachers asked to do?
• Review 3 pieces of student written work
• Choose 1 student’s strategy for a 1-on-1 conversation
• Describe an anticipated conversation & rationale (in writing)
Try It!

1-on-1 Conversations

**Problem:** The teacher has 4 pancakes to share equally among 6 children. How much pancake does each child get?

- Imagine that you are the teacher and would like to have a one-on-one conversation with Joy.

- Describe some ways you might respond to her work on this problem and explain your reasoning.

Joy’s strategy
The teacher has 4 pancakes to share equally among 6 children. How much pancake does each child get?

- Divided first 3 (or 4) pancakes into 4ths
- Passed out 1/4 to each child twice
- Divided last pancake into 8ths (likely cutting 4ths in half)
- Passed out 1/8 to each child
- Divided the last 2/8 into 6 pieces
- Passed out 1/24 to each child
- Answer
  - Wrote fractions as words
  - Drew unclear picture after “total”

Joy’s Strategy
How did we characterize teachers’ anticipated conversations?

• **Holistically:** To what extent did the teachers anticipate building on Joy’s thinking?

• **Teachers’ Anticipated Moves:** How closely connected were the teachers’ anticipated moves to Joy’s current strategy?

**INSIDE MOVES**

Teachers’ anticipated moves explore strategy details *inside* the child's current strategy

**OUTSIDE MOVES**

Teachers’ anticipated moves explore strategy details *outside* the child’s current strategy [new strategy]
Continuum of Building on Children’s Mathematical Thinking

- **Lack of Evidence**
  - No mention of the child’s mathematical thinking or mentioned to take it over

- **Limited Evidence**
  - General response; ask about child’s mathematical thinking but in minimal or non-specific ways

- **Robust Evidence**
  - Rich use of details of the child’s mathematical thinking; elicits and builds on specific thinking of the child
Range of Responses in the Continuum of Building on Children’s Mathematical Thinking

- Lack of Evidence: 14%
- Limited Evidence: 57%
- Robust Evidence: 12%
- Mixed Evidence: 17%
Robust Evidence: Ms. Starr’s Anticipated Conversation

• Can you tell me what you did?  
  (To understand the thinking behind the work)

• Why did you split the first 3 pancakes into 4 pieces?  
  (To understand the rationale, to see if she saw the relationship with the people)

• Tell me about the last pancake.  
  (I want to see what she was thinking when she split this pancake.)

• You wrote here 1/24. Can you show me 1/24 in the picture? How do you know that is 1/24?  
  (What thinking was behind this decision to split the pieces? What understanding does she have about it?)

• Do you know how much the kids will get altogether?  
  (Can she add her pieces?) Is it more than 1/2 or less? More than 1 or less?

• Is there another way to split the pancakes?  
  (Does she see the connection now?)
Robust Evidence: Ms. Starr’s Anticipated Conversation

• Can you tell me what you did? (To understand the thinking behind the work)
• Why did you split the first 3 pancakes into 4 pieces? (To understand the rationale, to see if she saw the relationship with the people)
• Tell me about the last pancake. (I want to see what she was thinking when she split this pancake.)
• You wrote here 1/24. Can you show me 1/24 in the picture? How do you know that is 1/24? (What thinking was behind this decision to split the pieces? What understanding does she have about it?)
• Do you know how much the kids will get altogether? (Can she add her pieces?) Is it more than 1/2 or less? More than 1 or less?
• Is there another way to split the pancakes? (Does she see the connection now?)
Lack of Evidence of Building on Children’s Mathematical Thinking

I would ask Joy if there was another way she could share 6 pancakes with 4 children. I would ask her if $\frac{1}{4} + \frac{1}{4} + \frac{1}{8} + \frac{1}{24}$ would be the most efficient way to share pancakes or if she could find a way to cut bigger servings.

She may have started with fourths without thinking about the outcome, so I would like to see if she could do thirds or halves. I know she understands equivalency so I would like to see what she (more efficiently) comes up with.
Lack of Evidence of Building on Children’s Mathematical Thinking

I would ask Joy if there was another way she could share 6 pancakes with 4 children. I would ask her if \(\frac{1}{4} + \frac{1}{4} + \frac{1}{8} + \frac{1}{24}\) would be the most efficient way to share pancakes or if she could find a way to cut bigger servings.

She may have started with fourths without thinking about the outcome, so I would like to see if she could do thirds or halves. I know she understands equivalency so I would like to see what she (more efficiently) comes up with.

2 total moves
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\frac{0}{2} = 0\% \text{ inside moves}
\]
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\frac{2}{2} = 100\% \text{ outside moves}
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## Lack of Evidence

I would ask Joy if there was another way she could share 6 pancakes with 4 children. I would ask her if $\frac{1}{4} + \frac{1}{4} + \frac{1}{8} + \frac{1}{24}$ would be the most efficient way to share pancakes or if she could find a way to cut bigger servings.

*She may have started with fourths without thinking about the outcome, so I would like to see if she could do thirds or halves. I know she understands equivalency so I would like to see what she (more efficiently) comes up with.*

## Robust Evidence (Ms. Starr)

- Can you tell me what you did? *(To understand the thinking behind the work)*
- Why did you split the first 3 pancakes into 4 pieces? *(To understand the rationale, to see if she saw the relationship with the people)*
- Tell me about the last pancake. *(I want to see what she was thinking when she split this pancake.)*
- You wrote here 1/24. Can you show me 1/24 in the picture? How do you know that is 1/24? *(What thinking was behind this decision to split the pieces? What understanding does she have about it?)*
- Do you know how much the kids will get altogether? *(Can she add her pieces?) Is it more than 1/2 or less? More than 1 or less?* - Is there another way to split the pancakes? *(Does she see the connection now?)*
Lack of Evidence

I would ask Joy if there was another way she could share 6 pancakes with 4 children. I would ask her if $\frac{1}{4} + \frac{1}{4} + \frac{1}{8} + \frac{1}{24}$ would be the most efficient way to share pancakes or if she could find a way to cut bigger servings. She may have started with fourths without thinking about the outcome, so I would like to see if she could do thirds or halves. I know she understands equivalency so I would like to see what she (more efficiently) comes up with.

Robust Evidence (Ms. Starr)

-Can you tell me what you did? (To understand the thinking behind the work)
-Why did you split the first 3 pancakes into 4 pieces? (To understand the rationale, to see if she saw the relationship with the people)
-Tell me about the last pancake. (I want to see what she was thinking when she split this pancake.)
-You wrote here $\frac{1}{24}$. Can you show me $\frac{1}{24}$ in the picture? How do you know that is $\frac{1}{24}$? (What thinking was behind this decision to split the pieces? What understanding does she have about it?)
-Do you know how much the kids will get altogether? (Can she add her pieces?) Is it more than $\frac{1}{2}$ or less? More than 1 or less?

-Is there another way to split the pancakes? (Does she see the connection now?)
Where do teachers focus their questions?

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Joy drew 4 pancakes first, then automatically cut them into fourths. She may feel comfortable with fourths? She knew the last one could be cut into sixths except for 1 fourth would be cut into twenty-fourths. She understands equivalence but cannot notate her thinking yet!

Could you re-create Joy’s strategy based on the description?

Joy drew her 4 pancakes and cut them into 1/4’s. I believe she did that because she is comfortable with 1/4’s. When she reached her last pancake she realized 1/4’s wouldn’t get each person a pancake piece. I think she then divided it into 1/8’s. Again, I think the 1/4’s and 1/8’s are comfortable for her. After she numbered 6 she realized she had 2 pieces left so she divided the last 2 1/8’s into 6 pieces. I believe she counted the pieces as if thirds were in each 1/8 to come up with twenty-fourths.
**Professional Noticing of Children’s Mathematical Thinking**

- Decomposition of responsive teaching (Grossman et al., 2009)
  - Professional Noticing—hidden, but critical part of responsive teaching expertise

- Importance of attending to strategy details
  - Linked to robust evidence of building on children’s thinking
  - Supports use of inside moves

- Anticipating 1-on-1 (circulating) conversations as a productive research & PD activity
  - Continuum of expertise -- window into how teachers intend to build (or not build) on children’s thinking during instruction
  - Circulating as a learning space of children and teachers
  - Heuristic for teachers for building on children’s thinking: inside/outside moves
Interested in learning more about mathematics teacher noticing?


Questions & Comments
Thank You!