SUPPORTING THE DEVELOPMENT OF TEACHERS’ EXPERTISE IN NOTICING CHILDREN’S MATHEMATICAL THINKING

Susan Empson
Vicki Jacobs
Joan Case
Dinah Brown

NCSM – April 2018
University of Missouri
Susan Empson

University of North Carolina at Greensboro
Vicki Jacobs
Naomi Jessup
Amy Hewitt

University of Texas at Austin
Gladys Krause
D’Anna Pynes

Other Partners: SRI & Teachers Development Group
PD Facilitators: Dinah Brown, Joan Case, & Luz Maldonado
This presentation is part of the CGI strand

• Cognitively Guided Instruction (CGI) is a research and teacher development program
In CGI classrooms…

• Teachers pose problems and children are encouraged to make sense of problems
• Children’s mathematical thinking is visible, valued, and is the basis for instructional decisions
• Teachers respond in the moment to children’s ever evolving thinking
Teachers respond in the moment to children’s ever-evolving thinking – Part 1

- 4th/5th grade
- Children are working on solving a fraction story problem

**Problem:** Kenzie loves to go on long hikes. She knows it’s important to drink water when she hikes. She drinks \( \frac{3}{4} \) cup of water for every mile she hikes. Her water bottle holds 4 cups of water. How many miles can she hike before her water runs out?

- Teacher circulates as children work
  - Stops to have a conversation with Mashanty
Teachers respond in the moment to children’s ever-evolving thinking – Part 1

- [VIDEO]
Teachers respond in the moment to children’s ever-evolving thinking – Part 1

• Ms. K: *Tell me about your picture.*

• Mashanty: *I was just thinking I think I did something wrong, because-*

• Ms K: *Why?*

• Mashanty: *I don’t get it. This picture’s not really working.*

Before teachers can respond to a child’s thinking they must **notice** the child’s thinking
Noticing Mashanty’s thinking so far

- Drew 4 cups of water as rectangles and partitioned each one into fourths
- Shaded 3/4 of each cup of water
- Felt unsure about whether her strategy would work
Professional Noticing of Children’s Mathematical Thinking

A child says or does something

The teacher responds

Noticing is what happens behind the scenes before the response
Professional Noticing of Children’s Mathematical Thinking

Three component skills of professional noticing of children’s mathematical thinking

1. **Attending** to the details of children’s strategies
2. **Interpreting** children’s understandings reflected in their strategies
3. **Deciding how to respond** on the basis of children’s understandings *(intended response)*
Guidelines for Supporting Teachers to Learn to Notice Children’s Mathematical Thinking

1) Use video & student written work to engage teachers in noticing without in-the-moment pressures

2) Emphasize the importance of attending to strategy details

3) Provide support for multiple years

4) Give teachers access to research-based knowledge of children’s mathematical thinking to enhance their noticing
USING VIDEO to engage teachers in noticing

Divine (4th grade)

What do you notice?

- How does Divine solve the problem? *(Attending)*
- What do you learn about Divine’s fraction understanding, based on her strategy? *(Interpreting)*
Divine (4th)  I made 9 little cakes and for every cake I made I used 2/3 cup of frosting, how much frosting did I use to make all of those cakes?

• [VIDEO]
Divine (4th)  I made 9 little cakes and for every cake I made I used 2/3 cup of frosting, how much frosting did I use to make all of those cakes?

- How did Divine solve the problem? *(Attending)*
- What did we learn about Divine’s fraction understanding? *(Interpreting)*
Noticing Divine’s Thinking

**Attending:** How did Divine solve the problem?

**PROBLEM:** 9 cakes, 2/3 cup of frosting on each

- Began by representing one cup of frosting, decomposed into 2/3 and 1/3
- Represented a second cup in the same way, then paused to silently count the number of groups of 2/3
- Continued to build up the groups of frosting, keeping track of the number and stopping when she got to 9
- Counted up the cups of frosting to get an answer of 6
Noticing Divine’s Thinking

Interpreting: What did you learn about Divine’s fraction understanding?

PROBLEM: 9 cupcakes, 2/3 cup of frosting on each

- Able to solve a multiplication of fractions problem by modeling the situation
- Able to distinguish between and keep track of two different wholes (a cake and a cup)
- Thinks flexibly about decomposing a whole into fractional quantities
  - Understands 1 cup can be decomposed into 2/3 cup and 1/3 cup
What do you notice?

• How did Divine solve the problem? *(Attending)*
  o What mathematical details are critical?

• What did we learn about Divine’s fraction understanding? *(Interpreting)*
  o What is your evidence?

• What are some possible responses to Divine? *(Deciding How to Respond)*
Noticing Divine’s Thinking

Deciding How to Respond: What are some possible responses to Divine?

PROBLEM: 9 cupcakes, 2/3 cup of frosting on each

• What questions might you ask:
  o To help you better understand Divine’s thinking?
  o To help Divine think about this problem and enhance her understanding?

\[
\frac{2}{3} \quad \frac{2}{3} \quad \frac{2}{3} \quad \frac{2}{3} \quad \frac{2}{3} \quad \frac{2}{3}
\]
Deciding How to Respond

• To help **you**:  
  o Why did you start by writing $\frac{2}{3} \quad \frac{1}{3}$?  
  o How did you know when all of your cakes were frosted?  

• To help **Divine**:  
  o Where are the 6 cups in your work?  
  o Where are the 9 cakes in your work?  

*Many possible good questions – Important thing is questions are based on Divine’s thinking and make room for her future thinking in the response*
USING STUDENTS’ WRITTEN WORK to engage teachers in noticing

Strategies for the snowcone problem (4th/5th grade)
Rosie has a snowcone maker. Each snowcone she makes uses 1/3 cup of ice. How many snowcones can Rosie make if she has 5 cups of ice?

• Take a look at the students’ written work
  o How did each child solve the problem? *(attending)*
  o What is revealed about the child’s fraction understanding, based on the strategy details? *(interpreting)*
Students’ written work: Strategy Sort

Order the set of work in a way that reflects the different levels of understanding that you saw in it.

What is your evidence?
- Which strategy details did you use to decide how to order?
Elijah’s Strategy

• Represents each snowcone and each cup individually

Rosie got a snow cone maker for Christmas. Each snow cone she makes uses \( \frac{1}{3} \) cup of ice. How many snow cones can Rosie make if she has 5 cups of ice?

Name: Elijah  Date: 1-24-14
Jasmine’s Strategy

- Represents each snowcone and each cup individually

Rosie got a snow cone maker for Christmas. Each snow cone she makes uses 1/3 cup of ice. How many snow cones can Rosie make if she has 5 cups of ice?
Samantha’s Strategy

• Works with the relationship between 1 cup and 3 snowcones, without representing every snowcone

• Builds up to 5 cups 1 cup at a time using this relationship
Rosie’s Strategy

- Works with the relationship between 1 cup and 3 snowcones, without representing every snowcone.
- Builds up to 5 cups 1 cup at a time using this relationship.

Rosie got a snow cone maker for Christmas. Each snow cone she makes uses $\frac{1}{3}$ cup of ice. How many snow cones can Rosie make if she has 5 cups of ice?
Rosie got a snow cone maker for Christmas. Each snow cone she makes uses $\frac{1}{3}$ cup of ice. How many snow cones can Rosie make if she has 5 cups of ice?

Andrew’s Strategy

- Works with the relationship between 1 cup and 3 snowcones, without representing every snowcone
- Builds up to 5 cups by multiplying
Selected for Range of Strategies

Elijah  Samantha
Jasmine  Rosie  Andrew
Noticing Mashanty’s thinking so far

- Drew 4 cups of water as rectangles and partitioned each one into fourths
- Shaded 3/4 of each cup of water
- Felt unsure about whether her strategy would work
Teachers respond in the moment to children’s ever-evolving thinking – Part 2

• [VIDEO]
What did you notice about Mashanty’s thinking?

- Drew 4 cups of water as rectangles and partitioned each one into fourths
- Shaded 3/4 of each cup of water
- Felt unsure about whether her strategy would work
- Knew that 3/4 cup of water was needed for Kenzie to go 1 mile
- Combined 3 of the extra 1/4s to make another 3/4 cup for another mile
Ms. K’s Noticing of Mashanty’s Mathematical Thinking

Mashanty says or does something → Ms. K responds

- Noticing
- Attends
- Interprets
- Decides
Guidelines for Supporting Teachers to Learn to Notice Children’s Mathematical Thinking

1) Use video & student written work to engage teachers in noticing without in-the-moment pressures

2) Emphasize the importance of attending to strategy details

3) Provide support for multiple years

4) Give teachers access to research-based knowledge of children’s mathematical thinking – noticing is a knowledge-intensive instructional practice.
Thank you!