Exploring Children’s Strategies for Equal Sharing Fraction Problems

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Responsive Teaching in Elementary Mathematics

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Other Partners: SRI & Teachers Development Group
Purpose of Session

- Explore children’s strategies for equal sharing problems with a fractional answer
- Introduce a summary chart on equal sharing strategies
- Explore different perspectives on children’s strategies of fractions
  - children’s thinking
  - traditional curriculum
What are Equal Sharing Problems?

Equal sharing problems are partitive division problems where the amount in each group is unknown.

<table>
<thead>
<tr>
<th>Total amount shared</th>
<th>÷</th>
<th>Number of groups</th>
<th>=</th>
<th>Amount in each group</th>
</tr>
</thead>
</table>

8 children shared 16 blocks of clay to do an art project. If they share the clay equally, how much clay would each child get?

16 ÷ 8 = 2

8 children shared 5 blocks of clay to do an art project. If they share the clay equally, how much clay would each child get?

5 ÷ 8 = 5/8
Solve It!

6 children are sharing 16 brownies so that everyone gets the same amount. How much brownie can each child have?

- If you were an upper elementary child, how would you solve this problem?
- Try to solve it two ways.
- Share your strategy with someone close to you.
Discussion

6 children are sharing 16 brownies so that everyone gets the same amount. How much brownie can each child have?

- How did you solve this problem?
Card Sort Activity

6 children are sharing 16 brownies so that everyone gets the same amount. How much brownie can each child have?

- Review the 11 pieces of student work
- Organize student work and keep track of decision making
  - Group similar strategies
  - Order from least to most sophisticated
- Keep track of features used to group and order strategies
Discussion: Card Sort

- What did you identify as your least sophisticated strategy? Why?

- What did you identify as your most sophisticated strategy? Why?
### Problem:
6 children are sharing 4 candy bars so that everyone gets the same amount. How much candy bar can each child have?

<table>
<thead>
<tr>
<th>Strategy Name</th>
<th>Strategy Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-Anticipatory Sharing</td>
<td>Child does not think in advance of both number of sharers and amount to be shared. For example, child splits each candy into halves because halves are easy to make. Gives each person ½ of a candy bar and a &quot;little piece.&quot; If the last candy bar is split, each person gets ½ of a candy bar.</td>
</tr>
<tr>
<td>Additive Coordination: Sharing One Item at a time</td>
<td>Child represents each candy bar. Splits first candy bar into sixths because that is the number of shares. Each person gets 1 sixteenth. Repeats process until all 4 candy bars are shared. Each person gets ½ of a candy bar altogether.</td>
</tr>
<tr>
<td>Additive Coordination: Sharing Groups of Items</td>
<td>Child represents each candy bar. Realizes that 6 pieces can be created by splitting 2 candy bars each into thirds. Each person gets ½. Child moves on to another group of items and continues similarly until all the candy bars are used up. Each person gets ½ of a candy bar altogether.</td>
</tr>
<tr>
<td>Ratio: Repeated Halving: Factors</td>
<td>Child may or may not represent all of the candy bars and people. Uses knowledge of repeated halving or multiplication factors to transform the problem into a simpler problem: 3 children sharing 2 candy bars. Solves the simpler problem. Each child gets ½ of a candy bar.</td>
</tr>
<tr>
<td>Multiplicative Coordination</td>
<td>Child does not need to represent each candy bar. Child understands that a things shared by 6 people is ½, so 4 candy bars shared by 6 people means each person gets ⅓ of a candy bar.</td>
</tr>
</tbody>
</table>

*Figure 11-17. Types of strategies children use to solve Equal Sharing problems*
Benefits of Frameworks about Children’s Thinking

- Supports teachers’ understanding of a range of strategies
- Highlights what children know based on their strategies
- Provides ways to support and extend children’s thinking
Framework for Equal Sharing Strategies

- Expanded from *Extending Children’s Mathematics*
- Three major distinctions of strategies

<table>
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<tr>
<th>Strategy Features</th>
<th>Non-Anticipatory (Direct Modeling)</th>
<th>Emergent Anticipatory (Direct Modeling)</th>
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<tbody>
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Framework for Equal Sharing Strategies

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Alicia (3rd grade)

Problem: 12 children want to share 3 burritos so that everyone gets the same amount. How much did each child get?
# Non-Anticipatory Direct Modeling

<table>
<thead>
<tr>
<th>Represents each share?</th>
<th><strong>Alicia</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>yes</strong></td>
<td></td>
</tr>
<tr>
<td>All partitions based on the number of sharers?</td>
<td><strong>not initially</strong></td>
</tr>
<tr>
<td>Creates equal shares and uses everything?</td>
<td><strong>yes</strong></td>
</tr>
<tr>
<td>Uses fraction words or notation?</td>
<td><strong>yes</strong></td>
</tr>
<tr>
<td>Combines unit fractions for a final share?</td>
<td><strong>not needed</strong></td>
</tr>
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**Problem**: 12 children want to share 3 burritos so that everyone gets the same amount. How much did each child get?
Non-Anticipatory Direct Modeling Strategies
6 kids share 16 brownies

Whole Items Only
- Passes out whole items only without any partitioning

Repeated Halving
- Starts partitioning by halving or repeated halving
- May finish partitioning based on number of sharers

Trial and Error
- Works through a small set of familiar fractions to determine which one results in partitioning items exhaustively

Erica

Janie

Daniel
Framework for Equal Sharing Strategies

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John (3rd grade)

**Problem:** 3 kids want to share 5 cookies. How much cookie would each kid get if they share equally?
Emergent Anticipatory Direct Modeling strategies

<table>
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<th>Represents each share?</th>
<th>John</th>
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<tbody>
<tr>
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<td>yes</td>
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<tr>
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**Problem:** 3 kids want to share 5 cookies. How much cookie would each kid get if they share equally?
Emergent Anticipatory Direct Modeling Strategies
6 kids share 16 brownies

Sharing One Item at a Time
- Partitions the first item into exactly as many parts as the number of sharers
- Repeats the process for each item until everything is shared

Sharing Groups of Items
- Uses multiplication facts or other number relationships to partition a group of items based on the numbers of sharers

Samantha

\[
\begin{array}{cccc}
1 & 2 & 3 & 4 \\
5 & 6 & 7 & 8 \\
9 & 10 & 11 & 12 \\
\end{array}
\]

Demarcus

\[
\begin{array}{cccc}
1 & 2 & 3 & 4 \\
5 & 6 & 1 & 2 \\
3 & 4 & 5 & 6 \\
2 & 3 & 4 & 5 \\
6 = 12 \\
123 & 456 & 123 & 456 \\
2 \text{ and } 2 \text{ thirds}
\end{array}
\]
# Framework for Equal Sharing Strategies

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</table>
Jordan (5th grade)

Problem: 6 kids are sharing 10 apple pies equally. How much does each child get?
## Anticipatory Equal Sharing Strategies

<table>
<thead>
<tr>
<th>Represents each share?</th>
<th>no</th>
</tr>
</thead>
<tbody>
<tr>
<td>All partitions based on the number of sharers?</td>
<td>yes</td>
</tr>
<tr>
<td>Creates equal shares and uses everything?</td>
<td>yes</td>
</tr>
<tr>
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<td>Combines unit fractions for a final share?</td>
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</table>

**Problem**: 6 kids are sharing 10 apple pies equally. How much does each child get?

Each child gets \(\frac{10}{6}\) of the pies, or \(1\frac{4}{6}\) of the pie.

\[1 + \frac{1}{2} + \frac{1}{6} = 1\frac{4}{6}\]
Anticipatory Equal Sharing Strategies
6 kids share 16 brownies

Multiplicative Coordination
- Mentally uses the relationship between number of sharers, number of items, and amount of items per sharer to determine a fractional outcome

16/6 since everybody gets 16 pieces and each piece is 1/6 of a brownie.

Keisha
## Overall Key Features

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<tr>
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<tr>
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<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Creates equal shares and uses everything?</td>
<td>Sometimes – not part of the initial plan but may get worked out along the way</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Uses fraction words or notation?</td>
<td>Sometimes but generally limited to a small set of familiar fractions</td>
<td>Usually</td>
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<tr>
<td>Combines unit fractions for a final share?</td>
<td>Sometimes with like unit fractions but rarely with unlike unit fractions</td>
<td>Usually with like unit fractions but only sometimes with unlike unit fractions</td>
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Unit Fraction Understanding

How is a unit-fraction quantity generally understood in these strategies?

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<tbody>
<tr>
<td>Part</td>
<td>Part</td>
<td>Number</td>
</tr>
<tr>
<td>No numerical relationship</td>
<td>Countable relationship</td>
<td>Multiplicative relationship</td>
</tr>
<tr>
<td>Combining unit fractions from multiple wholes is problematic</td>
<td>Can combine unit fractions from multiple wholes</td>
<td>Can combine unit fractions from multiple wholes and one is a referent unit</td>
</tr>
<tr>
<td>Represents every share?</td>
<td>Non-Anticipatory (Direct Modeling)</td>
<td>Emergent Anticipatory (Direct Modeling)</td>
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<td>-------------------------</td>
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### Typical Strategies

1. **Whole items only** — Child only distributes whole items without any partitioning
2. **Repeated Halving** — Child begins partitioning by halving or repeated halving without taking the number of sharers into account
   - If there are leftover parts or items, child may finish partitioning by taking the number of sharers into account
3. **Trial and Error** — Child works through a small set of familiar fractions to determine which one results in partitioning items exhaustively
   - Little or no attention to the

### How is a unit-fraction quantity understood in these strategies? (How do children see the part, the whole, and the relationship between the two?)

A unit-fraction quantity is a part contained within a whole, and the part does not necessarily have a numerical relationship to a whole. Instead, parts of any size can be called “pieces” or “halves” and counted without regard to their sizes.

A unit-fraction quantity is a part with a numerical relationship to a whole — a part fits into a whole a countable number of times. Parts (of the same size) from multiple wholes (of the same size) can be combined to make quantities greater than a whole.

A unit fraction is a number that refers to a quantity and is determined by the numerical relationship between a part and a whole — a unit fraction can be multiplied by the number of parts in the whole to make 1. Neither the part nor the whole needs to be whole.
**NON-ANTICIPATORY (DIRECT MODELING) STRATEGIES FOR EQUAL SHARING WITH FRACTIONS**

- Represents every share
- Begins partitioning without considering the number of sharers
- Sometimes creates equal shares and uses everything but not part of the initial plan—gets worked out along the way
- Sometimes uses fraction words or notation, but they are generally limited to a small set of familiar fractions
- Sometimes combines like unit fractions for a final share but rarely does so with unlike unit fractions

## SAMPLE STRATEGIES

**Problem:** 6 children are sharing 16 brownies so that everyone gets the same amount. How much brownie can each child have?

### Whole Items Only

**Distributes whole items only without any partitioning**

- Student draws 6 children and passes out 1 brownie to each child until the brownies run out and can no longer be given out fairly.

**“Each kid gets 2 brownies. The rest can be given to the teacher.”**

**“Everyone has 2 brownies. The rest are extras since there isn’t enough for everyone to have 1 more.”**

### Repeated Halving

**Begins partitioning by halving or repeated halving and, if necessary, finishes by taking the number of sharers into account**

- Student partitions all brownies in half and then partitions the last brownie into fourths and then eighths. After giving each child an eighth (“1 piece”), the remaining amount is partitioned into 6 pieces and each child gets 1 twenty-fourth (“1 little piece”).

**Student passes out 1 brownie to each child and partitions the remaining 4 brownies into fourths. After passing out 12 fourths to the 6 children, the remaining brownie is ignored because everyone will not be able to get a piece.**

### Trial and Error

**Works through a small set of familiar fractions to determine which one results in partitioning items exhaustively**

- Student passes out whole brownies to each child. Student then partitions the remaining brownies into halves but is unable to share them equally. Student then tries fourths but has the same problem. Finally, the student tries thirds and is able to share the brownies evenly.
Benefits of the Framework

- Supports teachers’ understanding of a range of strategies
- Highlights what children know based on their strategies
- Identifies different strategies and representations that reflect similar understanding
- Provides ways to support and extend children’s thinking
Big Ideas

- Pose equal sharing problems
- Engage with children’s fraction thinking
- Use the research-based frameworks to guide your work
- Be patient with yourself 😊
Questions?

Feel free to contact us for additional questions

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- Naomi Jessup
  njallen@uncg.edu

- Vicki Jacobs
  vrjacob@uncg.edu